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THE FAILURE OF OHM'S LAW IN GOLD AND SILVER AT HIGH CURRENT DENSITIES.

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Introduction.

It is to be expected that at high current densities the usual linear relation between current and E.M.F. in a metal will break down, that is, that Ohm's law will fail. J. J. Thomson,[1] for instance, has shown that on the basis of the classical free electron theory of metallic conduction the current will eventually increase only as the square root of the E.M.F. at very high values, which means that the resistance will increase at high current densities. On the usual assumptions the current densities at which this effect will become important are of the order of 10^{11} amp/cm^2. Many attempts have been made to detect the existence of this effect experimentally, but hitherto without success. The chief obstacle to success is that the changes of resistance due to heating by the heavy current are sufficient to mask the changes of resistance due to a possible departure from Ohm's law.

By the employment of a new method of attack, which avoids errors due to temperature rise, I have, I believe, not only succeeded in establishing the existence of the effect, but in measuring the departures from Ohm's law with some exactness in gold and silver. These results are described in the following paper. I find a departure from Ohm's law in the direction of an increase of resistance at high current densities, the maximum effect being of the order of 1% at a current density of 5×10^6 amp/cm^2.

HISTORICAL SURVEY.

A few attempts were made to detect the existence of the effect before Maxwell, but we may pass these over as not approaching in sensitiveness the method of Maxwell.[2] Maxwell was the chairman of a committee, the other members of which were J. D. Everett and A. Schuster, appointed by the British Association in 1876, to investigate the accuracy of Ohm's law. Two experimental methods were used, both apparently proposed by Maxwell; the experiments were performed by Chrystal.

The first was a substitution method, by which the resistances of five similar coils were compared in various combinations of two in parallel and two in series against a single one. The current density in the single coil was thus double that in the multiple arrangement, and if there is an effect of the kind sought, the resistances should be different. A small positive result was found, but was ascribed to errors in view of the fact that the second method, which was much more delicate, gave negative results.

The current densities employed in the first method were so low as not to cause appreciable heating of the wires. The second method was such that currents large enough to heat the wires to incandescence could be used. A heavy and weaker current were passed alternately in rapid succession through a fine wire, which was made one of the arms of an ordinary bridge. The period of alternation was so high that there were no appreciable cooling effects. Observations were made with a galvanometer of period long compared with that of the alternations. The apparent resistance was read first with the large and the small currents flowing in the same direction, and then with the small current reversed. Let us suppose that there is an effect of the kind sought, which means that the resistance for the large and the small current is not the same. If the galvanometer indicates balance, it must be because it is really off balance for both currents, to the one side for the large current, and to the other side for the small current. If now the small current is reversed the galvanometer will be off balance to the same side for both currents, and there will be a steady deflection. Hence if there is an effect, the balance will be altered by changing the direction of the small current. There were difficulties in the method. The period of alternation had to be chosen as high as 60 per second in order to avoid appreciable cooling effects in the short intervals of time when the small current replaces the larger one. There were considerable mechanical difficulties in designing an alternator of the requisite constancy, for it is obvious that the durations of the large and the small currents must be absolutely constant. A platinum contact dipping into a mercury cup and driven by a tuning fork was used, but always gave trouble, and the limits of accuracy were set by this part of the apparatus.

The conclusion as usually quoted which was drawn from these experiments was that any deviation from Ohm's law must be less than one part in 10^{12}. This statement needs some expansion; it is obvious that no measurements can be made directly to this degree of accuracy. For one thing, changes of temperature of the surroundings absolutely preclude the direct attainment of any such accuracy as this. The meaning of the statement is as follows. Maxwell remarked that any departure from Ohm's law must involve only even powers of the current; it is obvious that the first power cannot enter, for if so there will be a dependence of resistance on the direction of the current, which cannot be the case in an isotropic material. The initial departures from the law may be supposed to be proportional to the square of the current density. The maximum current density employed by Maxwell was 5.6×10^4 amp/cm^2. At this density the resistance was found to have changed by not as much as 0.3%. Assuming the square law, this means that at a current density of 1 amp/cm^2 the departure from Ohm's law cannot be more than 1 part in 10^{12}. The original paper contains the careful statement of the conclusion in this form.

The metals used by Maxwell were cylindrical wires of platinum (0.042 mm. diameter), German silver (0.051 mm.) and iron (0.14 mm.). The maximum current densities were respectively 3.4, 1.2, and 5.6×10^4 amp/cm^2.

Recently Wenner[3] of the Bureau of Standards has objected to the second form of Maxwell's experiment. He has repeated a modification of the first method with very much higher accuracy, and finds no deviation of as much as 3 parts in 10^7. His objection to the second experiment is that negative results might be obtained even if there is a departure from Ohm's law. Thus if the potential difference across each arm of the bridge is proportional to the square of the current flowing through it, negative results will be obtained, because the bridge will stay in balance for any current, large or small. More generally, negative results will be found if the potential difference across each arm is the same function of the current for each arm. It is to be noticed, however, that this is not the manner of departure from Ohm's law which is to be expected. The departure sought for is not a function of the total current flowing through the resistance, but is a function of the current density. Furthermore, this function is known to be nearly independent of current density for low values. Doubling the total current in a conductor of small section will produce a much greater departure from Ohm's law than doubling the same current in a conductor of larger section. This is the only sort of departure from Ohm's law which we are looking for, or indeed which seems at all likely, and in my opinion Maxwell's experiment is entirely competent to answer this question up to its limits of error. As regards the other sorts of departures from Ohm's law, I believe that Wenner's position is sound.

Since Maxwell, very few attempts have been made to detect the effect, probably because of the appalling sensitiveness of Maxwell's experiment as usually quoted. Lecher [4] in 1906 made measurements on platinum and silver wires. He attempted to correct for the temperature effect in the platinum by observing the thermal expansion of the wire when carrying a very heavy current, and comparing the resistance to this heavy current with the resistance to a feeble current passing through the wire when heated artificially to have the same thermal expansion as when carrying the heavy current. The diameter of the wire was 1 mm., and the current density was 3.8×10^3 amp/cm^2; there was no effect greater than 0.1%. The silver wire was 0.03 mm. in diameter. It was placed in a rapid stream of water, and the resistance measured under a current increasing until it fused. No temperature correction was applied. The apparent temperature when the wire burned out, using the ordinary temperature coefficient of resistance, and assuming that Ohm's law is true, was 130°. It is to be expected that this temperature would be somewhere between 100° and the melting point of silver. Hence within limits of error which may be several hundred percent, the experiment is consistent with Ohm's law. The accuracy is very much less than that of Maxwell, but on the other hand, the current density is very much higher, reaching a maximum of 1.4×10^6 amp/cm^2, 25 fold greater.

H. Rausch von Traubenberg [5] has attempted to avoid the temperature difficulty by employing a condenser discharge of very short duration. The current densities that may be reached are higher than previously realized, attaining in one case a maximum of 10^7 amp/cm^2. But the measurements of potential are inaccurate, being estimated by the break-down of a spark gap in air, and there are other sources of error arising from distributed capacities and inductances and the necessity of a long range extrapolation. I estimate that the error may certainly be as high as 10 or 15%. Within these limits, no effect was found. The material used was constantan, of a diameter of 0.1 mm. The maximum current density reached produced a potential gradient in the wire of 400 volts per cm. With "Kruppin" wire, a gradient of 1500 volts/cm. was reached, although with not so high a current density. In spite of the very considerable error, these experiments have pushed the boundary of the validity of Ohm's law considerably farther back than the previous limit. With regard to the materials used, it should be said that the maximum departure from the law is to be expected in those materials which, other things being equal, have a long free electronic path, and which are presumably the best conductors. From this point of view the most promising place to look for the effect is in silver, copper, and gold. Of course, on the other hand, it is to be said that in poor conductors it is possible to reach much higher potential gradients for the same current density, so that this advantage may outweigh the disadvantage.

OUTLINE OF METHOD.

In the method which I have used, the specimen exposed to the high current density is made one arm of a bridge, and its resistance is measured simultaneously for a heavy direct current and a small superposed sinusoidal current of acoustical frequency. If there is a deviation from Ohm's law the resistances to the two currents will not be the same. The following considerations show why this is. It must in the first place be remembered that a bridge is an instrument for testing the equality of potential of two points in a net work. In Figure 1, x denotes the arm of the bridge which contains the fine wire in which the

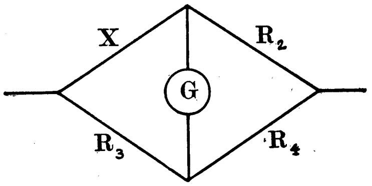

FIGURE 1. Skeleton of the bridge connections.

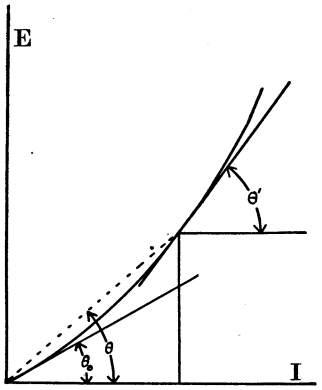

FIGURE 2. Hypothetical relation between current (I) and e.m.f. (E) not satisfying Ohm's law.

current density is to be high. In this branch we suppose that Ohm's law does not hold, but the relation between current and E.M.F. is given by a curve of the form shown in Figure 2. The other arms of the bridge, R_2, R_3, and R_4 are made of larger wire, in which the current density is always small, and hence their resistance is ohmic. Assume for the moment that it is possible to balance the bridge simultaneously for D.C. and A.C. Now let a heavy direct current I_1 flow through x and R_2 and a direct current I_3 through R_3 and R_4. Also let a small alternating current $i_1 \sin \omega t$ flow through x and R_2 and $i_3 \sin \omega t$ flow through R_3 and R_4. The difference of potential between the ends of x is

$$I_1 \tan \theta + i_1 \tan \theta' \sin \omega t.$$

The potential difference across the galvanometer is

$$(I_1 \tan \theta + i_1 \tan \theta' \sin \omega t) - (I_3 + i_3 \sin \omega t) R_3.$$

Since the extremities of R_2 and R_4 join, their potentials are equal, or

$$I_1 (\tan \theta + R_2) + i_1 (\tan \theta' + R_2) \sin \omega t = (I_3 + i_3 \sin \omega t)(R_3 + R_4).$$

This splits into two equations

$$I_1 \tan \theta + I_1 R_2 = I_3 (R_3 + R_4) \tag{1}$$

$$i_1 \tan \theta' + i_1 R_2 = i_3 (R_3 + R_4) \tag{2}$$

Now if the galvanometer is balanced for D.C. the constant part of the potential difference across its terminals must vanish, or

$$I_1 \tan \theta - I_3 R_3 = 0.$$

Combined with (1) above we get

$$I_1 R_2 - I_3 R_4 = 0,$$

or, dividing

$$\frac{\tan \theta}{R_2} = \frac{R_3}{R_4}. \tag{3}$$

In the same way, if the bridge is balanced for A.C. the alternating part of the potential difference across the galvanometer vanishes, or

$$i_1 \tan \theta' = i_3 R_3.$$

Combined with (2)

$$i_1 R_2 = i_3 R_4$$

and dividing

$$\frac{\tan \theta'}{R_2} = \frac{R_3}{R_4}. \tag{4}$$

Now conditions (3) and (4) are incompatible unless $\tan\theta' = \tan\theta$. Except for singular points, this means that the relation between current and E.M.F. in the arm x must be linear, or Ohm's law is satisfied. Conversely if Ohm's law is not satisfied, the setting for balance will not be the same for D.C. and small A.C., $\tan\theta$ may be called the direct current resistance, and $\tan\theta'$ the alternating resistance. They may both be determined by the ordinary bridge formulas by first adjusting R_3 and R_4 for D.C. balance, and then readjusting them for A.C. balance. The departure from Ohm's law at a given current density, which I denote by D, is the fractional difference between $\tan\theta$ and $\tan\theta_0$, the tangent to the curve at the origin, that is, the resistance under small currents. This definition gives the equation for D:

$$D = \frac{\tan\theta - \tan\theta_0}{\tan\theta_0}.$$

It is now obvious that if we measure θ and θ' at all points of the curve we can find the curve itself by an integration, hence the tangent at the origin, and so the departure from Ohm's law at any given current density. The mathematical details of this deduction will be given later.

It is evident that the method in simple outline, as given above, avoids the difficulty of the unknown temperature correction because both currents are flowing simultaneously, and hence the temperature of the wire is the same to both. There is, however, a temperature effect of a different kind from that usually met in this sort of experiment which arises as follows. The total rate of heat input under the current is proportional to the square of the total current, that is to $(I_1 + i_1 \sin \omega t)^2$. The $2I_1 i_1 \sin \omega t$ term in this expression denotes an alternate heating and cooling, so that superposed on the large steady temperature increase there is a small sinusoidal fluctuation of temperature whose average is zero. But this small fluctuation of temperature produces a small fluctuation of resistance, and a heavy current flowing through a fluctuating resistance gives rise to a fluctuating difference of potential at the terminals of the resistance. There is, therefore, effectively introduced into the x arm of the bridge a spurious additional sinusoidal E.M.F. which changes the A.C. balance. The action is similar to that of a microphone.

We now discuss mathematically this spurious E.M.F. and the experimental means taken to eliminate its effects. We are for the present concerned solely with this effect, and in the following treat the resistance as ohmic. Any residual effect left after the elimination of this "microphone action" constitutes the departure from Ohm's law for which we are searching. Various tacit assumptions will be made in the course of this discussion which will be justified later.

Return to Figure 1 for the bridge, and consider the heating effect in the arm x, treating its resistance as ohmic. The heat input is proportional to $(I_1 + i_1 \cos \omega t)^2$, where i_1 is small compared with I_1. Expanding this, neglecting the term in i_1^2, the rate of heat input is proportional to $I_1^2 + 2I_1 i_1 \cos \omega t$, that is, there is a constant input proportional to I_1^2, independent of the presence of the A.C., and there is a sinusoidal heating and cooling of the same period as the A.C. which is proportional in intensity to both the D.C. and the A.C. Under this heat input the conductor experiences a change of temperature, which may be analyzed into a constant change dependent only on the D.C., and a small alternating rise and fall, of the same period as the A.C., but not necessarily in phase with it. The factor of proportionality which determines the amplitude of the alternating part is not the same as that which determines the amplitude of the steady part, but is a function of the period, becoming less for higher frequencies. Let us call the steady change of temperature τ_0, the amplitude of the in-phase part of the alternating part τ_1, and that of the out-of-phase part τ_2. If the heat input is removed rapidly, τ_2 will be small compared with τ_1. The increase of temperature above that of the surroundings is therefore $\tau_0 + \tau_1 \cos \omega t + \tau_2 \sin \omega t$. Now if R_0 is the initial resistance at the temperature of the surroundings, α the temperature coefficient of resistance, and R the actual resistance when the current is passing, we have

$$R = R_0[1 + \alpha\,(\tau_0 + \tau_1 \cos \omega t + \tau_2 \sin \omega t)].$$

The potential difference across the terminals of x is

$$R(\mathrm{I}_1 + i_1 \cos \omega t).$$

Expanding this by substituting the value of R above, and using the relations

$$\cos^2 \theta = \tfrac{1}{2}\,(1 + \cos 2\theta), \quad 2 \sin \theta \cos \theta = \sin 2\theta,$$

we get:

$$\begin{aligned}\text{Potential difference} = \quad & R_0\{I_1(1 + \alpha\tau_0) + \tfrac{1}{2}\, i_1\alpha\tau_1\} \\ & + R_0\{I_1\alpha\tau_1 + i_1(1 + \alpha\tau_0)\} \cos \omega t \\ & + R_0\{I_1\alpha\tau_2\} \sin \omega t \\ & + R_0\{\tfrac{1}{2}\, i_1\alpha\tau_1\} \cos 2\omega t \\ & + R_0\{\tfrac{1}{2}\, i_1\alpha\tau_2\} \sin 2\omega t.\end{aligned}$$

Using the same notation as before for the current in the other arms of the bridge, we have at D.C. balance

$$R_0 I_1 \left\{ 1 + \alpha\tau_0 + \tfrac{1}{2}\alpha\tau_1 \frac{i_1}{I_1} \right\} = I_3 R_3,$$

and

$$I_1 R_2 = I_3 R_4.$$

Dividing to eliminate the currents, we obtain

$$R_0 \left\{ 1 + \alpha\tau_0 + \tfrac{1}{2}\alpha\tau_1 \frac{i_1}{I_1} \right\} = R_2 \frac{R_3}{R_4}.$$

This shows that in general, even neglecting the $\cos^2$ term in the heat input as we have above, the D.C. balance will depend on the A.C. But this effect is doubly small, since τ_1 is small compared with τ_0 and i_1 small compared with I_1, and hence the effect may be neglected. The correctness of this assumption was checked experimentally.

With regard to the A.C., the expression above shows that there cannot be complete balance. There will always be higher harmonics, and there will be an out-of-phase component (in $\sin \omega t$). These terms are small, as examination of the coefficients shows, but may nevertheless be perceptible. The ear can set on the fundamental alone, and so eliminate the higher harmonics. The out-of-phase component gives rise to a smearing out of the sharpness of the minimum. This can be corrected by introducing another out-of-phase component to neutralize it by a variable mutual inductance between input and detecting circuits.

The equilibrium conditions for the in-phase component are

$$R_0 i_1 \left[1 + \alpha\left(\tau_0 + \tau_1 \frac{I_1}{i_1}\right) \right] = i_3 R_3 .$$

and

$$i_1 R_2 = i_3 R_4.$$

Eliminating the current,

$$R_0 \left[1 + \alpha\left(\tau_0 + \tau_1 \frac{I_1}{i_1}\right) \right] = R_2 \frac{R_3}{R_4}.$$

The condition for A.C. balance is therefore different from that for D.C. balance, the large term I_1/i_1 occurring in the expression for A.C. balance against the small term i_1/I_1 in the expression for D.C. balance.

As the experiment was actually performed, R_2 was kept constant, and R_3 and R_4 were varied. R_3 and R_4 consisted of extension coils connected to a bridge wire, which was tapped by a moving slider. Hence adjustment was made by adding to R_3 an appropriate resistance, and at the same time subtracting the same resistance from R_4.

Let us call the initial resistance for balance with no heating effect (small D.C. or A.C. only) R_3 and R_4. To maintain balance under the heavy D.C. with no A.C., R_3 must be increased by ΔR and R_4 diminished by ΔR. A.C. balance with the heavy D.C. flowing is now maintained by an additional increase of $\Delta R'$ to R_3 and decrease of R_4 by $\Delta R'$. The conditions for balance under these three states of current flow are:

$$R_0 = R_2 \frac{R_3}{R_4} \tag{5}$$

$$R_0\left[1 + \alpha\left(\tau_0 + \tfrac{1}{2}\tau_1 \frac{i_1}{I_1}\right)\right] = R_2 \frac{R_3 + \Delta R}{R_4 - \Delta R} \tag{6}$$

$$R_0\left[1 + \alpha\left(\tau_0 + \tau_1 \frac{I_1}{i_1}\right)\right] = R_2 \frac{R_3 + \Delta R + \Delta R'}{R_4 - \Delta R - \Delta R'} \tag{7}$$

Subtracting (6) from (7) and discarding squares and products of ΔR and $\Delta R'$ gives

$$R_0\alpha\tau_1\left[\frac{I_1}{i_1} - \tfrac{1}{2}\frac{i_1}{I_1}\right] = R_2 \frac{\Delta R'(R_3 + R_4)}{R_4^2 - R_4\,(2\Delta R + \Delta R')}.$$

Also neglecting $\tfrac{1}{2}i_1/I_1$ compared with I_1/i_1, and substituting for R_0 its value from (5) gives

$$\alpha\tau_1 = \frac{i_1}{I_1} \frac{\Delta R'\left(1 + \frac{R_4}{R_3}\right)}{R_4 - (\Delta R' + 2\Delta R)},$$

which gives again approximately

$$\alpha\tau_1 = \frac{i_1}{I_1}\,\Delta R'\left(\frac{1}{R_3} + \frac{1}{R_4}\right). \tag{8}$$

From (6), for the D.C. setting, we get approximately

$$\alpha\tau_0 = \Delta R\left(\frac{1}{R_3} + \frac{1}{R_4}\right) \tag{9}$$

Hence finally we have

$$\frac{\tau_1}{\tau_0} = \frac{i_1}{I_1}\,\frac{\Delta R'}{\Delta R}. \tag{10}$$

If now the alternations are slow, so that at every moment the wire is approximately in thermal equilibrium, the factor of proportionality connecting τ_1 with the alternating heat input is the same as that connecting τ_0 with the steady heat input, so that we would have

$$\left.\begin{array}{l}\tau_0 = \text{const } I_1^2 R \\ \tau_1 = \text{const } 2I_1 i_1 R\end{array}\right\}$$

which gives, substituting above in (10),

$$\frac{\Delta R'}{\Delta R} = 2.$$

That is, for slow alternations, the difference between A.C. and D.C. settings due to the microphone action alone is twice the D.C. shift due to temperature rise under the steady current, and this relation holds no matter how feeble the alternating current. Since the steady rise of temperature is high, because the current density has to be pushed to the limit that the conductor will carry without burning out, it is obvious that at slow alternations the microphone action will entirely mask any sought for deviation from Ohm's law. The acoustical frequencies used in these experiments were not low enough to reach the extreme value 2 for the ratio $\Delta R'/\Delta R$. At the lowest frequency, 320 cycles, the ratio had reached about 1.2.

At rapid rates of alternation, however, the conditions of heat transfer change. At low frequencies the thermal conductivity of the surroundings alone determines the equilibrium; at higher frequencies part of the heat input is used in raising the temperature of the surroundings and a term enters proportional to the specific heat, and at still higher frequencies this term preponderates, and the factor of proportionality between amplitude of rate of heat input and amplitude of temperature alternation becomes proportional to the specific heat and inversely proportional to the frequency. We shall later apply a dimensional analysis to obtain more information about τ_0 and τ_1, but for the present we may write, for any frequency

$$\tau_1 = \text{const} f(\omega)\, 2I_1 i_1 R,$$

and as before

$$\tau_0 = \text{const } I_1^2 R.$$

This now gives

$$\frac{\Delta R'}{\Delta R} = 2f(\omega).$$

At high frequencies $f(\omega) \propto \frac{1}{\omega}$, so that at high frequencies the microphone effect is proportional to the reciprocal of the frequency, and it may be eliminated by proceeding to infinite frequencies (or zero reciprocal frequency).

The procedure suggested by this analysis was that followed in the experiment. For a fixed D.C. the difference between the settings at D.C. and A.C. balance (that is $\Delta R'$) was observed over a range of frequencies, $\Delta R'$ was plotted against the reciprocal of frequency, and extrapolated to zero. The residual, if there is one, is the effect due to deviation from Ohm's law at the particular D.C. density in question.

This procedure was repeated for a number of currents, and so the departure from Ohm's law obtained as a function of current density.

Before proceeding further with the theoretical discussion it will pay now to describe the experimental details, in order that we may have an idea of the order of magnitude of the quantities involved.

EXPERIMENTAL DETAILS.

The bridge was an ordinary four gap alternating current bridge, so constructed that inductive and capacity effects in the bridge were negligible. The resistance R_2, which was kept constant during the measurements on any single specimen, was a coil of heavy manganin wire immersed in an oil bath to carry away the Joulean heat. This resistance was approximately equal in magnitude to the resistance x which carried the high current density. The resistances R_3 and R_4 consisted of heavy manganin coils connected by a slide wire, which was tapped by a slider. The wire was about 1 meter long, with a total resistance of about 3 ohms. The resistance of the extension coils was five or ten times greater than that of the specimen x, and the generation of heat in them was so small that it was not necessary to immerse them in an oil bath.

The method of connecting the D.C. and the A.C. sources and of tapping across with the detectors for D.C. and A.C. balance is shown in Figure 3. The direct and alternating current sources are connected to the same terminals of the bridge, with a large inductance L in the D.C. line to prevent the A.C. backing into the battery, and a large condenser C in the A.C. line to prevent the D.C. backing into the A.C. source. D.C. balance was shown by a Leeds and Northrup high sensitivity galvanometer of about 8 ohms internal resistance connected as shown with a high resistance R_7 in series and another small resistance in shunt to cut down the sensitivity. In the latter part of this work this galvanometer was replaced by another of less sensitiveness. The A.C. detector was a telephone tapped between the same points as the D.C. detector, but with a large condenser in series to prevent D.C. getting into the telephone circuit. The telephone was tapped across a transformer placed in this circuit. In this circuit is also one of the coils of a mutual inductance, M, the other coil of which is in series with the A.C. source, and is not shown. This makes possible the elimination of the out-of-phase component by suitable adjustment. The A.C. was prevented from entering the galvanometer circuit by the high resistance in series with it, and by an open key when the galvanometer was not in use. The condenser in the telephone circuit proved an unnecessary precaution, the resistance of the transformer and mutual inductance being sufficient to prevent enough diversion of the D.C. into the telephone line to introduce appreciable error. The condenser was used in most of the work, but in some of the later readings it was omitted. The telephone was one of 1100 ohms resistance, made by the Western Electric Co., type 509 W. The transformer was one of the small ones of the General Radio Co. made for this purpose, type 166.

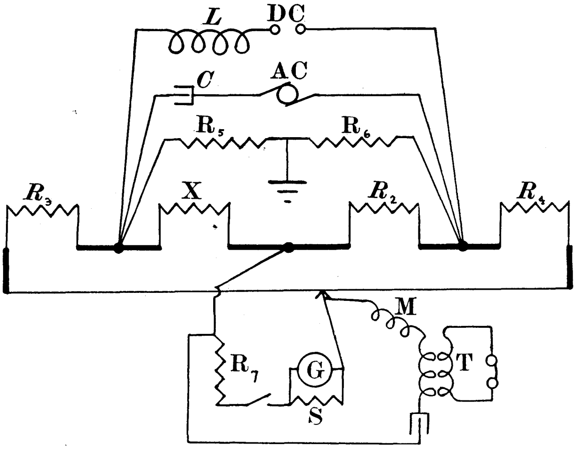

FIGURE 3. Details of the bridge connections.

In Figure 3, the resistances R_5 and R_6 which are connected to the same points as the sources of the current constitute an auxiliary bridge.

The intermediate point was put to ground. This is the regular method of avoiding capacity effects in the telephone. R_5 and R_6 were so large that there was no serious diversion of current from the bridge.

The D.C. source was a storage battery connected in series with a General Electric Co. ballast lamp (iron filament in hydrogen) of 1.4 amp. capacity. In series with the lamp was a commercial ammeter with which the constancy of the input current was checked. Any desired fraction of the output of the battery could be diverted from the bridge by a variable shunt between the lamp and the bridge. The actual current into the bridge or through the sample was not measured directly, but was computed from the ammeter reading, and the resistances, which were measured with the requisite accuracy. If a heavier current than 1.4 amp. were needed, two ballast lamps could be used in parallel. The dimensions of the sample were such that in almost all cases the maximum current that it could carry without burning out was not over 1 amp.

The source of A.C. was a vacuum tube oscillator. The methods of connecting this were the canonical methods, and need not be gone into here. I am much indebted to Professor L. E. Chaffee and Mr. S. Ballantine for assistance and advice in setting up this circuit. For the first readings a Western Electric Co., hot lime transmitting tube Type VT2 was used, but this soon was burned out, and for most of the work a G. E. transmitting tube, type T Pliotron, was used.

Not only does the present differ from preceding attempts in the method of measurement, but also in the form and dimensions given to the metallic resistance carrying the high current density. In all preceding work the metal has been in the form of a fine wire, of diameter of the order of 0.001 inch or more. An elementary discussion will show that a wire of these dimensions will carry only a limited current of the order of 10^7 amp/cm^2 in the most favorable case. The limit is reached at a rate of heat input so high that the interior of the wire is at the melting point while the outer surface is at 0°, the thermal conductivity of the metal just sufficing to carry off the heat input under the temperature gradient so produced. It is possible to gain somewhat by rolling the wire flat as a galvanometer suspension, but not a great deal. An elementary dimensional discussion will show that the only way to gain on the upper limit of current density is by decreasing the thickness of the specimen, so that a given difference of temperature between interior and exterior will give a larger temperature gradient, and therefore a greater heat dissipation. The thinnest metal that can be obtained is in the form of beaten leaf, and it was with gold and silver leaf that I made the measurements. I was afraid to try films deposited by cathode spattering because it did not seem to me that the condition of the metal was sufficiently like that of ordinary metals, whereas the leaf may be supposed to be more like the massive aggregates of metal of ordinary dimensions. However, a few experiments at the end with spattered films of gold gave the same results as beaten leaf of the same thickness, and my fears are probably ill founded. The next work that suggests itself in this connection is an extension of the results found here for silver and gold to other metals, using spattered films.

The thickness of the gold and silver leaf was determined by weighing a known area, assuming in the calculation that the density is the same as that of ordinary metal. The thickness of the silver leaf was 2.0×10^{-5} cm. Three thicknesses of gold were used, 8×10^{-6}, 1.67×10^{-5}, and 5×10^{-5}.

It was with some difficulty that I obtained the intermediate thickness of gold. Gold is beaten out in comparatively large quantities at a time in books of gold beaters skin, a great many thicknesses together. The last stage of the beating reduces the thickness by a factor of 6, from 5×10^{-5} to 8×10^{-6}, and only these thicknesses can be obtained commercially. I am indebted to Mr. Drew of Province Court, Boston, for his kindness in interrupting the last stage of the beating, and at some trouble removing a few of the partially beaten leaves from a large book. The sheets so obtained from the partially completed process were not nearly as perfect as those from the normally completed beating.

The state of the metal in a thin film differs in some unknown respects from that in larger masses. It has long been known that the specific resistance of spattered films is several times higher than that of the massive metal.[6] The specific resistance of gold films has been shown to be very high for very small thicknesses, to decrease rapidly as the thickness increases up to a certain point where the resistance is about five times normal, from here on to remain nearly constant over a range of thickness of about 20 fold, and beyond this point to decrease to the normal value. The temperature coefficient of resistance of spattered films has been frequently observed to be negative. The films of leaf metal used in this work did not show such great abnormalities as the usual spattered films, but nevertheless the resistance was very different from that of the massive metal. The temperature coefficient of my gold leaf was about 0.0015 between 0° and 30°, and was the same for the two thicknesses with which most of the measurements were made, namely 8×10^{-6} and 1.67×10^{-5} cm. The temperature coefficient of the silver was much more nearly normal, and was 0.0032, its thickness being 2.0×10^{-5}. The specific resistance of these metals was much higher than normal. That of the thinnest gold varied from 8.4 to 19.7×10^{-6} ohms per cm. cube, average 11.6. The spattered films of the same thickness varied from 15.4 to 23.8×10^{-6}, average 19.2. The normal value for massive gold is 2.42×10^{-6}. The thicker gold had a higher resistance than the thinner, varying from 9.75 to 18.5, average 13.3×10^{-6}. The cause of the high specific resistance of the spattered gold is doubtless to be found partly in the lack of crystalline structure and perfect coherence, due to its manner of formation. The high resistance of the leaf metal, on the other hand, is doubtless in large part due to mechanical imperfections. Examination of the thinnest leaf under a microscope shows a large number of folds and creases; it is practically impossible to spread the leaf on a surface so that it will lie smoothly in a single unwrinkled layer. The mechanical imperfections in the thicker gold were even greater than in the thinner, as already mentioned, doubtless partly due to the interruption of the beating process at a disadvantageous stage. Two samples of gold 5×10^{-5} thick had resistances of 15.0 and 10.0×10^{-6}, not essentially different from the other pieces.

The specific resistance of the silver leaf varied from 3.5 to 5.1×10^{-6}, average 4.1×10^{-6}. The normal value for silver is 1.63×10^{-6}. It is seen that silver lies much closer to the normal than does gold. Under the microscope it too was full or minute imperfections, but of a different character from the gold. There were no folds, but a number of minute round perforations through the leaf.

It would doubtless have been most desirable if these experiments could have been performed on more massive samples with the normal electric constants, but the necessity of conducting away the heat seems absolutely to preclude such a possibility.

The resistance has to be artificially cooled if current densities high enough to obtain an appreciable effect are to be reached, and the problem of mechanical support had to be solved. For this purpose the leaf metal was mounted on a piece of glass. The glass was covered with a very thin coat of insulating enamel by dipping it in a very dilute solution of the enamel in chloroform, the leaf metal was blown or otherwise spread over the surface, and was then baked at 210° until the enamel was hard. Gold or silver leaf so attached to the surface of glass is full of minute flaws, but by a search under the microscope parts can usually be found of sufficient homogeneity. The leaf was cut so as to leave a narrow isthmus of the shape shown in Figure 4. It is the isthmus that carries the high current density. Connection to the leaf on either side of the isthmus was by means of fine leads of copper caught to the leaf with a touch of solder. A special tool had to be made for cutting the isthmus. The point of a very fine needle was made to travel in any desired direction across the surface of the foil, scratching through to the glass, by an arrangement of two screws at right angles to each other. With this device, under the lens of a microscope, the isthmus could be cut to a high degree of precision. The dimensions of the isthmus varied somewhat from specimen to specimen, but the length was of the order of 1 mm. and the width of the order of 0.1 mm. The dimensions of each specimen were measured with a microscope.

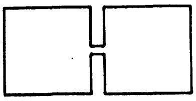

FIGURE 4. The isthmus form of the specimen.

The specimen was cooled by a stream of water flowing across the isthmus at right angles to its length. This water was delivered from a small glass nozzle suitably held and directed. At first kerosene was used as a cooling liquid, in order to avoid danger of short circuit, but the cooling was not sufficiently rapid and the desired current densities could not be reached. I also tried currents of compressed air and hydrogen, with results very much inferior to those even for kerosene. In order to protect the specimen from the short circuiting action of the water, it was covered on the upper surface, except over the isthmus itself, with an additional coating of enamel. Any enamel on the isthmus itself is fatal. At first I used tap water, but this was too conducting. Ordinary distilled water, however, proved to be sufficiently insulating so that no short circuiting effects from it could be detected. After the distilled water had been used for some time slight irregularities began to appear due to increasing conductivity from miscellaneous impurities picked up from the air of the room; these irregularities could be made to disappear by replacing the water with fresh.

It is necessary that the velocity of the cooling water be maintained constant. For small streams, a syphon arrangement was satisfactory, but for more rapid delivery the proper head was maintained by air pressure obtained from a large compressed air bottle, and was regulated to any desired value with a safety valve of special construction.

In addition to the gold and silver leaf, I made attempts to detect the effect in manganin wire 0.001 inch thick rolled flat, and with Wollaston wire of platinum about 0.00006 inch thick. The attempt with manganin failed because the heating effects were too large, due to the dimension of the specimen. The attempt with platinum failed because of mechanical difficulties in mounting the wire and subjecting it to a stream of water. It is possible that with more pains it might be feasible to obtain results with platinum in this way.

The thickness of the leaf metals used in this experiment was more than sufficiently small to ensure conduction of the Joulean heat developed by the heavy current without excessive rise of temperature. To illustrate the order of magnitudes involved let us consider an example. One specimen of silver that gave good results had the following dimensions: Length 0.536 mm., width 0.072 mm., and thickness 2×10^{-5} cm. The maximum current before the specimen burned out was 0.745 amp., and the initial resistance was 1.30 ohms. The heat input into this specimen was therefore:

$$\text{Heat input} = \frac{(.745)^2 \times 1.30}{4.18} \text{ gm cal/sec.}$$
$$= 0.173 \text{ gm cal/sec.}$$

This heat flows out through the area of one face, which is $0.0536 \times 0.0072 = 3.87 \times 10^{-4}$ cm^2. The heat outflow per unit area is therefore $(0.173)/(3.87 \times 10^{-4}) = 4.5 \times 10^2$ cal/sec cm^2. Since the thermal conductivity of silver is approximately unity, the temperature gradient required to drive this thermal stream is 4.5×10^2 degrees per cm. But the total thickness of the film is 2×10^{-5}, so that the extreme temperature difference in the specimen between front and back face is $4.5 \times 10^2 \times 2 \times 10^{-5} = 0.009°$.

It is of interest to compare the heat input with the heat capacity of the specimen. Its volume is $3.87 \times 10^{-4} \times 2 \times 10^{-5} = 7.8 \times 10^{-9}$ cm^3. Taking for the specific heat of silver 0.056, and the density as 10.5, we find the heat capacity to be $10.5 \times 0.056 \times 7.8 \times 10^{-9} = 4.6 \times 10^{-9}$. If there were no heat outflow the temperature would rise at the rate of $(0.173)/(4.6 \times 10^{-9}) = 0.038 \times 10^9 = 38{,}000{,}000$ degrees per second.

The magnitude of the steady temperature rise actually observed in this specimen was about 50°, or 5000 times more than the mean rise of temperature required to procure conduction of the heat input out of the metal itself. It is obvious, therefore, that practically all the resistance to heat out flow is in the thin layer of cooling water immediately in contact with the surface of the metal. Our previous estimate of the maximum current density that a wire can carry must be cut down many fold. A 0.001 inch wire cannot carry 10^7 amp/cm^2 under practical conditions. It still remains true under these new conditions, however, that the only change of dimensions of the specimen which will increase the maximum obtainable density is a decrease of diameter.

Our numerical example shows that the body of the metal can be regarded as approximately at a single constant temperature, both for the steady rise of temperature and for the alternating fluctuations. In the mathematical discussion above it was assumed that the temperature of the metal could be specified by a single number; the numerical discussion just given constitutes the justification of this.

DIMENSIONAL DISCUSSION OF THE COOLING PROCESS.

In order to get further in our understanding of the phenomena we must now consider in some detail the steady and alternating changes of temperature τ_0 and τ_1, remembering that practically all the resistance to heat outflow is in the cooling water. It is of course not possible to give an exact solution; the best that we can do is to give a dimensional discussion. Let us consider in the first place the equations of heat transfer in a fluid that is in motion. The equations may be obtained by a slight generalization of the process by which the equation of heat transfer is deduced for a medium at rest. Let us suppose that the medium is homogeneous except for temperature differences, that its specific heat per unit volume is c and its velocity of motion at any point v. Consider a small closed surface S at any point in the liquid. The rate of rise of temperature of the matter within this surface is the total heat input divided by the heat capacity. The heat input consists of two parts. The first is the ordinary conduction across the boundary, and is $\int\int k \frac{\partial \tau}{\partial n} dS$, where k is the thermal conductivity. This assumes that the velocity v is so small compared with the velocities of molecular motion within the liquid that the ordinary process of conduction takes place independent of the motion. The second part of the heat input is that which is convected, and is $-\int\int c\tau v_n dS$. From these two expressions we get the equation

$$\int\int\int c \frac{\partial \tau}{\partial t} dv = \int\int \left(k \frac{\partial \tau}{\partial n} - cv_n\right) dS.$$

Applying Green's theorem to the surface S, transforming the surface to volume integrals, using the condition that $\mathrm{Div}\, v = 0$ because the liquid is to be considered as incompressible, and removing the integral sign, gives for the differential equation of heat transfer

$$\frac{\partial \tau}{\partial t} = \frac{k}{c} \nabla^2 \tau - v \cdot \mathrm{Grad}\, \tau .$$

This equation applies at points inside the liquid. There will also be an equation to fix the boundary conditions. This equation is of the ordinary type, independent of the motion of the liquid, and is merely the statement that the heat input across the boundary is equal to the conductivity of the fluid multiplied by the normal temperature gradient.

Apply these equations now to the present problem. If the motion of the liquid is not turbulent, and if the lines of flow are not altered by the heat input, then at the surface of the metal the liquid flows in planes parallel to the surface, the velocity increasing from the surface. The determination of the velocity distribution is a problem of hydrodynamics, and involves the viscosity of the liquid and the variables which describe the mechanical roughness of the surface, but as far as we are interested in the problem the elements which enter our heat equations are determined if we can specify the velocity gradient at the surface. The other elements which enter the equation of heat transfer are the thermal conductivity of the liquid and its specific heat per unit volume.

Subject now to the restrictions mentioned, we may make a dimensional analysis of the situation. Notice in the first place that since the flow of water is transverse to the specimen the rise of temperature etc. is independent of the length, provided only that the specimen is long enough for us to neglect end effects.

We now enumerate the elements with which we are concerned and their dimensions.

Name of Quantity	Symbol	Dimensional Formula
Average rise of temperature	τ	τ
Rate of heat input per unit length	Q	$H L^{-1} T^{-1}$
Velocity gradient in liquid	g	T^{-1}
Thermal conductivity of liquid	k	$H L^{-1} T^{-1} \tau^{-1}$
Specific heat of liquid per unit volume	c	$H L^{-3} \tau^{-1}$
Breadth of specimen	b	L
Frequency of impressed heat input	ω	T^{-1}

Consistently with our numerical discussion we have not tabulated the thermal properties nor the thickness of the specimen itself, since the effect of these is vanishingly small.

We have now two cases to consider; first the steady temperature rise. The period of the impressed heat input does not enter, and we have to find all the dimensionless products of the first six quantities of the list above. Since there are four fundamental kinds of quantity (instead of unit quantity of heat H, we might have expressed heat in mechanical units, thus replacing H by M, with no change in the final result), and hence two dimensionless products. Inspection shows these products to be $k\tau/Q$ and k/gcb^2. Hence the relation which we want may be expressed as

$$\tau = \frac{Q}{k} f\left(\frac{cgb^2}{k}\right),$$

where f is some unknown arbitrary function. This relation can be tested by experiment, and so some idea obtained of the correctness of the assumptions underlying the discussion. For instance, at constant rate of flow of cooling water, the above equation shows that the steady temperature rise should be proportional to the rate of heat input, or to I^2. We can obtain an additional check for low rates of flow. For low rates, but not too low, it seems natural to assume that an important part of the rise of temperature is inversely proportional to the rate of flow, or inversely as g. This means that in f there is an important term which is the reciprocal of its argument, and we obtain as a partial expression

$$\tau = \text{Const}\; Q/gcb^2.$$

Some experimental information may be obtained here by varying the breadth of the sample at approximately equal rates of flow. That the average rise of temperature should be less for the greater breadth seems somewhat paradoxical, and affords a more drastic test than the proportionality of temperature rise to the rate of heat input.

Now let us consider the alternating fluctuations of temperature. To distinguish from the steady case, and consistently with the previous notation, we denote the amplitude of the alternating heat input by Q_1, and the amplitude of the alternating temperature change by τ_1. We now have seven quantities, and hence three dimensionless products. The additional product, beside the two already obtained, is g/ω. The relation between the variables now takes the form

$$\tau_1 = \frac{Q_1}{k} \varphi\left(\frac{cgb^2}{k}, \frac{g}{\omega}\right),$$

where φ is an arbitrary unknown function of its two arguments. For high frequencies g/ω is a small quantity. It is obvious in the first place that at high frequencies the temperature amplitude cannot approach a constant value, but must vanish. Hence developing the function φ for small values of the argument g/ω, putting the constant term zero, and retaining only the first order terms, we have for high frequencies the approximate relation

$$\tau_1 = \frac{Q_1}{k} \cdot \frac{g}{\omega} \cdot \psi\left(\frac{cgb^2}{k}\right).$$

If now we make the further assumption that the conductivity cannot enter at high frequencies, the effect being determined by the specific heat alone, we get the approximate relation

$$\tau_1 = \text{Const}\ \frac{Q_1}{c\omega b^2}.$$

The striking thing about this relation is that the velocity of flow has disappeared, the amplitude of temperature fluctuation being proportional to the heat input, and inversely as the frequency. This again is a result that can be tested by experiment, and its verification would go far toward making probable the assumptions underlying the discussion.

These values which we have found by dimensional analysis for the steady and alternating changes of temperature may now be substituted back in the relation $\tau_1/\tau_0 = i_1\Delta R'/I_1\Delta R$ which we obtained from the equations for the bridge. The rate of heat input entering the dimensional formulas for τ_0 and τ_1 may be written down at once in terms of the currents, namely

$$Q = I^2{}_1\, R'$$
$$Q_1 = 2I_1 i_1\, R',$$

where R' is the resistance per unit length. Substituting these values in the dimensional formulas for τ_0 and τ_1, and these again in the value of τ_1/τ_0 above, gives

$$\frac{\Delta R'}{\Delta R} = 2\, \frac{\varphi\left(\frac{cgb^2}{k}, \frac{g}{\omega}\right)}{f\left(\frac{cgb^2}{k}\right)}.$$

We now discuss the various experimental checks which were made of the adequacy of the apparatus and the correctness of the assumptions underlying the mathematical discussion. One may perhaps feel that I have gone too much into detail here, but in view of the importance of the result, and the feeling of disquiet which must be produced by the fact that the final result has to be obtained by an extrapolation I do not believe that multiplication of precaution is superfluous.

VARIOUS EXPERIMENTAL CHECKS AND PRECAUTIONS.

Relative Magnitude of D.C. and A.C. A fundamental requirement of the experiment has been that the A.C. be small compared with the D.C. If it is not, the $\cos^2$ term in the heat input becomes appreciable, and the steady temperature rise is different when the A.C. is flowing than when it is absent. This is easy to check experimentally. If the A.C. is sufficiently small, the D.C. setting should be the same whether the A.C. is flowing or not. This condition was checked repeatedly. Before the experiments were begun the constants of the oscillating circuit were so chosen for the different frequencies that the condition should be met. In general the accuracy of setting for D.C. zero was much greater than for A.C. zero. The latter could not usually be set much closer than 0.5 mm. of bridge wire, whereas the D.C. balance could be determined to 0.1 mm. Sometimes at the lower frequencies there was a displacement of the D.C. balancing point by an appreciable amount when the A.C. was turned on, but less than the error in the A.C. reading. Error at low frequencies does not affect the extrapolation. At the high frequencies, however, any shift of D.C. balance in the presence of the A.C. was less than 0.1 mm. An error of 0.1 mm. in the slider setting means on the average an error of 1/20000 in the resistance.

An additional check was afforded by the requirement that the difference between A.C. and D.C. settings (that is $\Delta R'$) should be independent of the A.C. intensity over a considerable range of A.C. intensity in the neighborhood of that used in the measurements. This requirement is shown at once by the formula on page 152. At constant D.C., i.e. constant ΔR, $\Delta R'$ is independent of the alternating current. This requirement obviously continues to be demanded if part of the $\Delta R'$ is due to a departure from Ohm's law of the kind for which we are searching, for this departure depends only on the D.C. density and not on the small A.C. This again was checked over the range of frequencies, with particular care at the three highest frequencies, as error here will affect the extrapolation. The check is not as good as that above because of the greater error of the A.C. readings, but the requirements were met within the errors of reading, which were 0.5 mm. The A.C. intensity was varied over a range of 10 or 20 fold, and its intensity determined by means of a thermo-couple. At the greater intensities of this range, the difference between A.C. and D.C. settings might be of the order of 1 cm.

For the actual measurements the A.C. was made as large as possible in conformity with these requirements, in order that the sensitiveness might be as high as possible. The requisite intensity varied somewhat at the different frequencies, (there was resonance at about 1000 cycles in the telephone) but as a rough average was about 0.02 amp. This is of the order of 1/30 of the D.C. current near its maximum, so that the $\cos^2$ term, which must be negligible, is of the order of 5×10^{-4} of the steady heating term. The D.C. shift of zero near the maximum was of the order of 15 cm. of bridge wire. $5 \times 10^{-4} \times 15$ is 0.075 mm., which is thus beyond the possibility of detection or error.

Equality of A.C. and D.C. Zeroes. If the bridge is properly set up and free from capacity and inductive effects, the D.C. zero for small direct currents and the A.C. zero (that is, the A.C. setting in the absence of D.C.) should be the same. This condition was satisfied within the limits of reading at the three highest frequencies. The settings could be made and read to 0.1 mm. for both A.C. and D.C. It was only when the A.C. settings were made in the presence of D.C. that there was room for as much uncertainty as 0.5 mm. At the lower frequencies the two zeroes might sometimes differ by as much as several tenths of a mm., but this does not affect the extrapolation or the final results.

Independence of Rate of Cooling. If the extrapolated difference between A.C. and D.C. readings is due to a real departure from Ohm's law, and is not in some way connected with the microphone action, which depends on the heating, then for a fixed D.C. density the extrapolated difference of readings must be independent of the steady rise of temperature, which may be made to vary by changing the speed of the cooling water. That this condition is satisfied is shown in Figure 5, in which the difference between D.C. and A.C. settings is plotted against the reciprocal of the frequency. The D.C. heating current was the same for the two curves, but the rate of flow of the cooling water was different, the steady rise of temperature of the upper curve being 1.4 times as great as that of the lower.

Effect of Speed of Cooling Water on Steady Temperature Rise. For low speeds of cooling water with a constant D.C. the rise of temperature decreases rapidly with increase of speed. At higher rates of flow, however, the effect becomes much less, until above a certain point there is very little dependence of steady temperature rise on the rate of flow. In the actual experiments the speed was chosen high enough to be within this insensitive range. This ensures that conditions on different days with different samples were rather closely the same. There were slight outstanding differences in the cooling conditions of the different samples, however, due to slight changes in the position of the sample in the holder, changes in the angle of presentation of the nozzle of the cooling stream, etc.

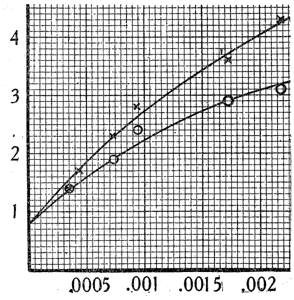

FIGURE 5. Results obtained with gold 8×10^{-6} cm. thick. The abscissae are reciprocals of the frequency and the ordinates difference between A.C. and D.C. settings in cm. of bridge wire. The two curves were obtained with the same D.C. but with different heating effects, produced by varying the rate of flow of the cooling water. The two curves should extrapolate to the same point if there is a genuine departure from Ohm's law.

Dependence of Steady Temperature Rise on Rate of Heat Input. Our dimensional analysis, subject to the restrictions assumed for the manner of flow of the cooling water, has shown that the steady rise of temperature does not depend on the metal, but only on its breadth, the rate of heat input, and several other factors which do not change if the cooling fluid is always water, and its velocity is in the range where it does not much affect the cooling. Now the rate of heat input is proportional to the square of the D.C. and may be computed in terms of the resistance and the dimensions of the specimen. This computation was made, and the results plotted for a dozen specimens, comprising gold of the three thicknesses and silver.

In the first place the steady temperature rise for all these specimens is a linear function of the rate of heat input up to rates which are near the burning out point. When this point is approached there is a break in the curve, and the temperature rise increases more rapidly than the heat input. The probable explanation is that the lines of flow of the cooling water are changed, it not being unlikely that there are localities where the water is even vaporized. The general magnitude of the temperature rise observed is consistent with this idea.

In the second place, the steady temperature rise shows no correlation whatever with the thickness of the sample or its material, but the rise for the thickest and the thinnest gold or for the silver is approximately the same for the same breadth of sample. This establishes the correctness of the assumption that the dissipation takes place in the layer of water in contact with the specimen, and does not depend at all on the properties of the metal, always provided of course, that the metal is sufficiently thin. The rise of temperature for specimens of different breadths is less at the same input per cm^2 of surface for the wider specimens. The total variation was by a factor of 2, the rise of temperature of the narrowest specimen per unit heat input per cm^2 being about twice that of the widest. The breadths varied from 0.007 to 0.022 cm. At the same time it was a matter of experiment that it was possible to reach higher current densities in the narrow samples without burning out than in the wider ones. The reason for this is that the break in the curve at which the linear relation between temperature rise and heat input ceases is reached much sooner for the wide than for the narrow samples.

Effect of Breadth of Sample on Heating. In order to test more exactly the precise dependence of rise of temperature on breadth, a series of runs was made on the same piece of gold, cutting down the breadth after each run so as to make it successively narrower. The sample was initially 1.18 mm. broad and 1.33 mm. long. End effects are important at these dimensions, and agreement with the results of the dimensional analysis is to be expected only for the narrower samples. Readings were made at six breadths. In the first place, as the breadth became less the heating effect became continually greater for the same current. Now if ρ is the two dimensional resistance of the thin metal film, and b its breadth, its resistance per unit length is ρ/b, and the heat input Q per unit length is $I^2\rho/b$. Our dimensional analysis shows that

$$\tau = \frac{I^2}{k} \cdot \frac{\rho}{b} \cdot f\left(\frac{cgb^2}{k}\right).$$

Now f at one extreme, for very slow rates of flow, contains an important term which is the reciprocal of its argument, whereas at high rates of flow we have seen that experiment suggests that it tends to become constant. Hence, keeping all the other arguments constant except b, the variation of τ may be expected to be between $1/b$ and $1/b^3$.

Experimentally the three narrowest breadths were 0.090, 0.155, and 0.305 mm. The shift of D.C. setting (proportional to temperature rise) for 0.2 amp. was respectively 8.5, 3.4, and 0.2 cm. Variation as $1/b^3$ would have given 8.5, 1.7, 0.3; as $1/b^2$, 8.5, 2.9, 0.9; and as $1/b$, 8.5, 5.0, and 2.7. The variation is within the limits set and probably nearly as $1/b^2$, within the limits of error.

This check is very rough, but does bear out the paradoxical variation with b given by the dimensional argument. The difficulties of the experiment are great, it being impossible to exactly reproduce the conditions of flow when the sample is removed from position, its breadth cut down, and then replaced. In the actual measurements of the departures from Ohm's law, this source of irregularity was of course not present, a complete series of measurements at different D.C. strengths being made on the sample unchanged in position and with the same flow.

These measurements also showed the same fact as that mentioned in the preceding section, namely that the relation of proportionality between heat input and temperature rise ceases at high rates of heat input, and the point of break in the linear relation comes at lower values of heat input per unit area for the large than the small breadths. This set of measurements with changing breadth of the same sample showed that the current at the point of break is roughly proportional to the square root of the width, which means that the break occurs when the generation of heat per unit length reaches a fixed value, independent of the breadth.

Dependence of Results on Position of Electrodes. There is another sort of check of entirely different character which may be made by changing the position of the electrodes on the sheet of metal leaf. There is an essential difference between the resistance of a two and a three dimensional mass of conducting material. If two electrodes are immersed in a three dimensional conducting medium, and the distance between them is increased indefinitely, the resistances between them will approach a finite value, the sum of the so-called electrode resistances. On the other hand in a two dimensional medium, the total resistance between electrodes of definite shape increases indefinitely as the distance between the electrodes is increased indefinitely. One cannot speak of the electrode resistance of a two dimensional conductor. Applied to the present case, this means that if the electrodes are not situated near to each other on opposite sides of the isthmus, the total resistance will not be merely the resistance of the isthmus and practically nothing else, but the rest of the sheet will make a finite contribution. But the rest of the sheet will not make a finite contribution to the departure from Ohm's law, since the departure increases very rapidly with increasing current density, and it is only in the isthmus that the density is high.

The experiment was tried of making a specimen with two sets of electrodes, one pair close to the isthmus, and the other pair much more remote. The total resistance between the second pair of electrodes was 1.27 times that between the first. Now if the effect found is genuine, the extrapolated value of $\Delta R'$ for the same value of the current density in the isthmus should have the same absolute value irrespective of the position of the electrodes, and should not merely be the same fraction of the total resistance between electrodes. That this demand is met is shown in Figure 6. The points for the two positions of the electrodes lie on the same smooth curve within the errors of the measurements, and this error is evidently much less than a factor of 1.27.

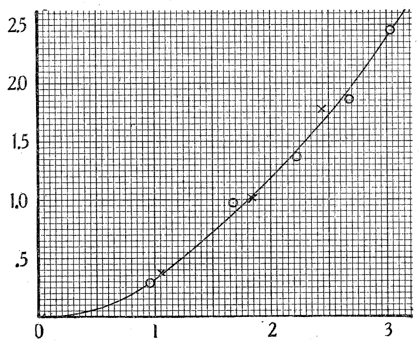

FIGURE 6. Results obtained with gold 8×10^{-6} cm. thick. Abscissae, current density in 10^6 amp/cm^2, ordinates, extrapolated difference between A.C. and D.C. settings in cm. of bridge wire. The two sets of points were obtained with the same specimen, but with different positions of the electrodes. The points shown by the crosses are the observed points corrected by the ratio of the total resistance to the resistance of the isthmus. If the effect measured is a genuine departure from Ohm's law, the corrected points should fall on the same curve with the others.

EXPERIMENTAL PROCEDURE AND DATA.

After the preparation of the sample, and measurement of its dimensions, it was adjusted in the holder, the fine copper wires which were soldered to the metal leaf were in turn attached to heavier leads, and these were connected to the proper gap in the bridge, and the nozzle for the cooling water was adjusted to give the most efficient stream over the specimen. As already mentioned, the cooling water was distilled water. This was contained in one of two large carboys, which were at the temperature of the room. On starting the water, there was no change in the zero due to change of temperature of the specimen. Because of the limited capacity of the carboys, it was necessary several times in the course of the experiment to interrupt the readings to turn water from one carboy into the other.

Readings were begun at some small value of the direct current, obtained by insertion of the proper shunt in the feeding circuit. An initial reading was made of the A.C. zero, with no D.C. flowing. Readings were now made of the D.C. balancing point with A.C. flowing, and the A.C. balancing point with D.C. flowing, and the two readings were repeated with D.C. reversed. This set of four readings was repeated for each of several frequencies, beginning at the lowest. The frequencies used were 320, 460, 680, 1140, 1530, 2450, and 3750. The frequencies were calibrated with tuning forks. At each frequency the mutual inductance was readjusted to give the sharpest setting.

Originally more readings were taken. These were the balancing point for small D.C., A.C. zero with no D.C., D.C. balancing point for large D.C. with no A.C. flowing, balancing point for large D.C. with A.C. flowing, and balancing point for A.C. with large D.C. flowing. After a number of runs had been made, taking all these readings, I was satisfied that these precautions were not necessary, and the readings mentioned above proved sufficient.

The A.C. zero, which gives the actual resistance of the specimen itself, may change slightly during a run at a single D.C. intensity and a series of frequencies. This may be due to some mechanical change in the specimen, because of friction by the cooling water, or at high current densities may be due to incipient burning out. Any such change during a run was always small however, and could be disregarded in the computations.

The series of readings at different frequencies was now repeated at some higher value of D.C., and the series continued until so high a

D.C. was reached that the specimen burned out. It was usual to make readings at five to seven different D.C. intensities.

The difference between D.C. and A.C. settings for each constant D.C. were now plotted against the reciprocal of the frequency, and the results extrapolated to zero (that is, infinite frequency). Of course the chief point of the experiment is this extrapolation. It is in the first place to be remarked that the extrapolation is not a wide one, the observed frequencies covering a range of ten fold, so that the distance of extrapolation is only one tenth of the distance covered by the readings. It is in the second place to be noticed that we have theoretical evidence as to the form of the curve in the neighborhood of zero. The part of the plotted difference which involves the heating effect, and which we are trying to eliminate varies linearly as the reciprocal of the frequency. The remainder, which is the departure from Ohm's law, is constant, independent of the frequency. The curves therefore extrapolate as straight lines, their curvature disappearing as they approach the axis. Given this condition, the extrapolation can be made with very little uncertainty. The curve is already so nearly straight at the last two or three frequencies that in most cases a straight line could be drawn through these points without error. However, this is not quite satisfactory. In the early part of the work I made the extrapolation by plotting the results on a large scale, and drawing free hand the curve which seemed to best satisfy the conditions. Later, however, in order to make the process of extrapolation more mechanical and less subject to error by personal bias, I adopted the following mechanical device. A steel spring was made to conform as closely as possible to the observed points by imposing on it three restrictions. It was made to go through, or near, the observed point at the lowest frequency, another point near the middle of the frequency range, and it was made to go through such a point on the axis that it coincided as closely as possible with the points at the upper end of the frequency range. This process will be more fully described in the next paragraph.

Three typical sets of readings, giving differences between A.C. and D.C. settings in terms of cm. of bridge wire plotted against reciprocal of frequency, are shown in Figures 7, 8, and 9. These figures are for the two thicknesses of gold and the one of silver. It will be seen that there can be little doubt about the extrapolation. The fact that the extrapolated curve does not pass through the origin, but strikes the axis at a point higher up, constitutes the evidence for the departure from Ohm's law. The mechanical extrapolation was made as follows.

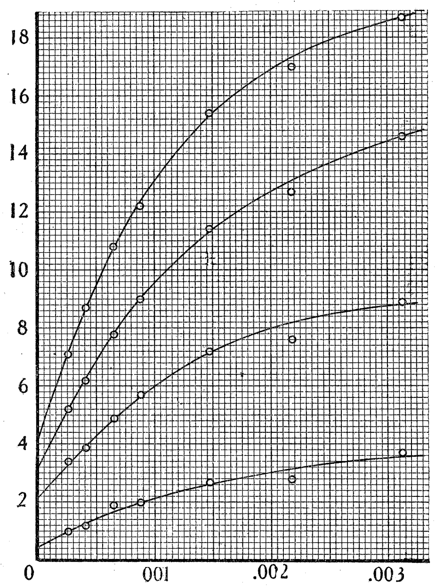

Fig. 7.

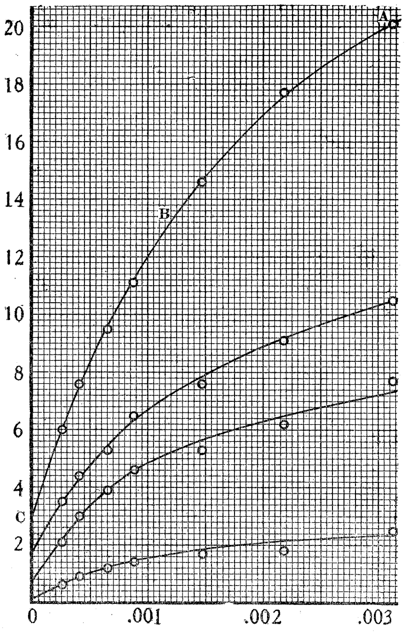

Fig. 8.

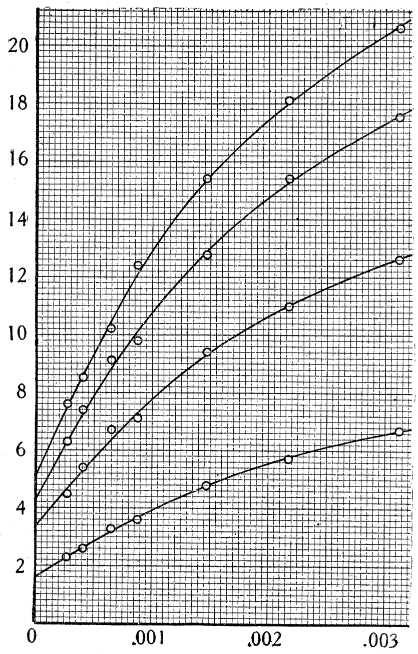

Fig. 9.

Figure 7. Results obtained with one specimen of gold 8×10^{-6} cm. thick. Abscissae, reciprocal of the frequency; ordinates, difference between A.C. and D.C. settings in cm. of bridge wire.

Figure 8. Results obtained with one specimen of gold 1.67×10^{-5} cm. thick. Abscissae, reciprocal of the frequency; ordinates, difference between A.C. and D.C. settings in cm. of bridge wire.

Figure 9. Results obtained with one specimen of silver 2×10^{-5} cm. thick. Abscissae, reciprocal of the frequency; ordinates, difference between A.C. and D.C. settings in cm. of bridge wire.

Consider the upper curve in Figure 8. A needle was stuck through the paper at the point *A* and another at *B*. The spring was passed under *A* and over *B*, and under a third needle *C* which was moved up and down the axis of ordinates at the same time that slight trial readjustments were made in the positions of *A* and *B* until that combination was found at which the spring coincided most closely with the observed points, particularly at the high frequencies. It will be seen that under these conditions the spring crosses the axis in a straight line, which is one of the demands. The spring also passes through *A* as a straight line, which it theoretically should not do. Any error due to this cannot have a perceptible effect on the extrapolated value. A number of times I made independent extrapolations by the free hand method and by the use of the spring, with essentially the same results.

Additional evidence of the correctness of the extrapolations is the fact that these were all made independently. The extrapolations were first made, and then afterwards the factors were computed by which the extrapolated differences in cm. of bridge wire were reduced to fractional parts of resistance. When the extrapolations were made I had no knowledge of whether the results would conform to the results with other specimens or not.

Successful results were obtained with gold 8×10^{-6} and 1.67×10^{-5} cm. thick, and with silver 2×10^{-5} thick.

The collected results for the eleven samples of 8×10^{-6} gold (that is, extrapolated difference between A.C. and D.C. settings as fractional parts of the initial resistance plotted against current density) are shown in Figure 10. The breadth of these samples varied from 0.066 to 0.226 mm., and both gold leaf and spattered gold are included in the results. If the effect is genuine, there should of course be no relation with the breadth. We have seen that there is a striking dependence of heating effects on the breadth, but it is evident that there is no such correlation here, and that within the limits of error points for all samples lie on the same curve. This, I believe, constitutes rather telling evidence for the genuineness of the effect. With regard to the magnitude of the experimental error, it is true that the points are perhaps more scattered than one could wish, but when we consider that we are dealing with an effect whose existence even has not hitherto been established, although much sought for, I do not believe that the scattering is more than one might expect.

The collected results for gold 1.67×10^{-5} thick are shown in Figure 11. This comprises results on four different samples. The results are much more irregular, and the limits of error are greater for the thicker than the thinner gold. The extrapolation is more uncertain because perceptible curvature continues to higher frequencies than for the thinner gold. It was not possible to reach such high current densi-

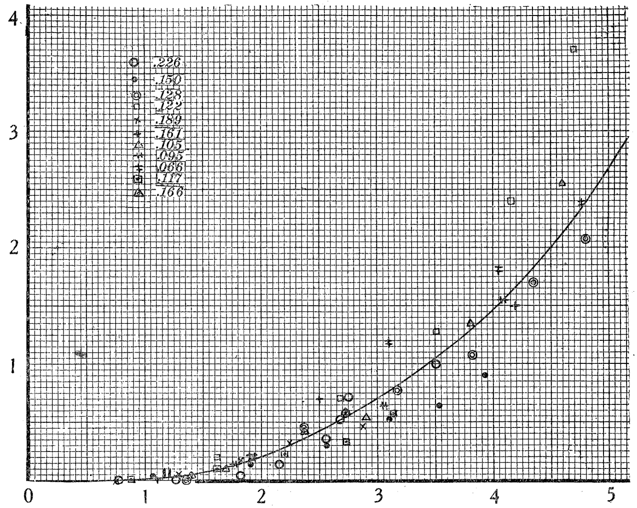

FIGURE 10. Collected results for gold 8×10^{-6} cm. thick. Abscissae, current density in 10^6 amp/cm^2, ordinates, extrapolated difference between A.C. and D.C. resistance in per cent of D.C. resistance. The numbers in the body of the diagram show the breadth of the various samples in mm.

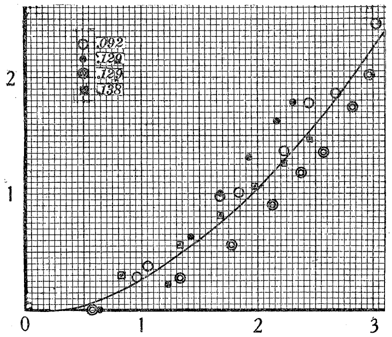

FIGURE 11. Collected results for gold 1.67×10^{-5} cm. thick. Abscissae, current density in 10^6 amp/cm^2; ordinates, extrapolated difference between A.C. and D.C. resistance in per cent of D.C. resistance. The numbers in the body of the diagram show the breadth of the various samples in mm.

ties without the specimen burning out. If the maximum current density without burning out is fixed by the amount of heat that can be carried away by the cooling water, and if this is independent of thickness, as we have seen it is approximately, then the maximum current density for a piece of twice the thickness should be inversely as $\sqrt{2}$. This relation is approximately satisfied. It is to be noticed that the effect is greater for the thicker gold; this would be expected from theoretical reasons.

In addition to these two thicknesses of gold, I tried several samples of the next size gold that I could obtain in the market, namely 5×10^{-5}. Results with this were unsatisfactory. The curvature of the relation between A.C. and D.C. difference and reciprocal frequency continues to much higher frequencies than for the thinner specimens, and in fact is quite marked over the entire frequency range, so that it was not possible to extrapolate. It can be seen at once that the inequalities of temperature in a leaf of five times the thickness are twenty-five times as great for the same current density.

The results for silver 2×10^{-5} cm. thick are shown in Figure 12. Measurements were made on five different samples, of widths varying from 0.072 to 0.165 mm. The results are more scattered than for the thin gold, but again there can be no question of the existence of the effect, and the fact that there is no correlation with the breadth.

In addition to these measurements on gold and silver I attempted to measure the effect in aluminum. The thinnest aluminum leaf that can be obtained in the market is 5×10^{-5} cm. thick, and I found the same trouble with it that I did with the thickest gold, namely that the curvature persists to such high frequencies that the extrapolation cannot be made with any assurance.

OTHER POSSIBLE EXPLANATIONS OF THE EFFECT.

Skin Effect. It is natural to search for explanations of these results other than a departure from Ohm's law. One of the first that suggests itself is the "skin effect." The resistance of a conductor is higher to alternating than direct currents because the alternating current tends to collect in the surface layers, not having time to soak into the interior of the conductor. This effect is very important at wireless frequencies with conductors of the ordinary dimensions. It increases rapidly with increase of frequency, and is greater in large than in small conductors. An upper limit for the conductor used in these experiments may be found by applying

Rayleigh's formula for a conductor of circular section to a conductor of the same area of section as that of the specimens actually used. At the upper limit of frequencies used here the skin effect for such a circular conductor is only one part in 10^{12}, and is hence absolutely not to be considered.

The Electrostriction Effect. The pinch effect is well known in conductors carrying heavy currents, and it might be expected that under the high current densities of these experiments there might be mechanical strains set up in virtue of the magnetic action of the current elements on each other which would alter the resistance. But it is to be noticed that the pinch effect depends on the total current carried by the conductor and not on its density. The currents carried here are only of the order of 1 ampere, and numerical calculation shows that any such effect is beyond the possibility of detection.

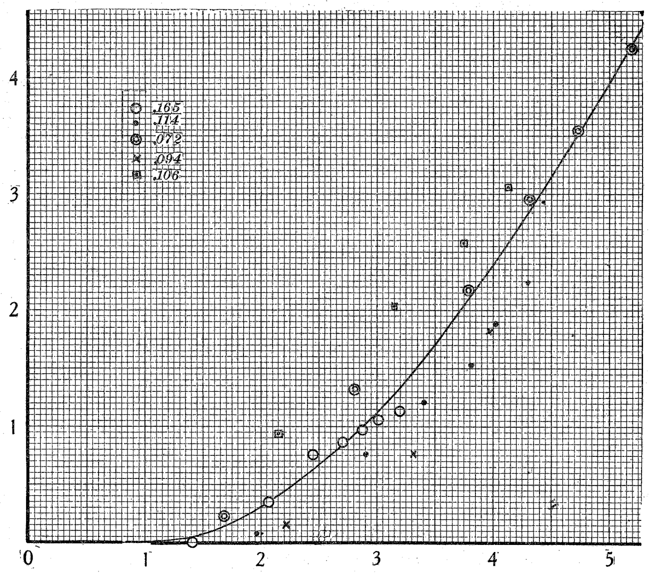

FIGURE 12. Collected results for silver 2×10^{-5} cm. thick. Abscissae, current density in 10^6 amp/cm^2; ordinates, extrapolated difference between A.C. and D.C. resistance in per cent of D.C. resistance. The numbers in the body of the diagram show the breadth of the various samples in mm.

Variation of Resistance with Frequency. The electron theory predicts a change of resistance at high frequencies independent of the skin effect. It is usually considered that this effect is important only at optical frequencies, but it is worth our while to consider it here. The formula may be found on page 432 of Richardson's book on Electron Theory of Matter, and is

$$\sigma_P = \frac{\sigma_0}{1 + p^2 \dfrac{m^2 \sigma_0{}^2}{N^2 e^4}}$$

Here σ_P is the conductivity at the frequency in question, σ_0 the conductivity at zero frequency, p the angular velocity corresponding to the frequency, m the mass of the electron, e the charge on the electron, and N the number of electrons per cm^3. The only quantity in this formula which we do not know definitely is N. Let us assume for the moment that N is equal to the number of atoms. If now we substitute numerical values for silver we find that σ_P differs from σ_0 by 5.3×10^{-18}. (I have used for $p = 2\pi \times 10^4$, which is more than twice the highest experimental value). But now according to my theory of conduction, the number of electrons must be very considerably less than the number of atoms, and this will increase the difference between σ_P and σ_0. If we assume as an extreme value that the number of electrons is 10^{-4} as great as the number of atoms, we find that σ_P still differs from σ_0 by 5.3×10^{-10} at 10,000 cycles. Evidently this effect is not a factor under our conditions.

Computation of the Departure from Ohm's Law.

Referring again to Figure 2, we have defined the departure from Ohm's law as $(\tan \theta - \tan \theta_0)/\tan \theta_0$. What we have actually measured and plotted in Figures 10, 11, and 12 is $(\tan \theta' - \tan \theta_0)/\tan \theta_0$. Given this as a function of current, we require to find the departure from Ohm's law as a function of current. For convenience replace E by y, and I by x. Let the required curve be represented by $y = f(x)$. Experimentally we determined

$$\frac{\dfrac{dy}{dx} - \dfrac{y}{x}}{\left(\dfrac{dy}{dx}\right)_{x=0}} = \varphi(x),$$

where φ is a known function. Without essential restriction we may put $\left(\frac{dy}{dx}\right)_{x=0} = 1$. We now have to solve

$$\frac{dy}{dx} - \frac{y}{x} = \varphi(x).$$

The solution is

$$y = x\left\{\int_0^x \frac{\varphi(x)\,dx}{x} + \text{const.}\right\}.$$

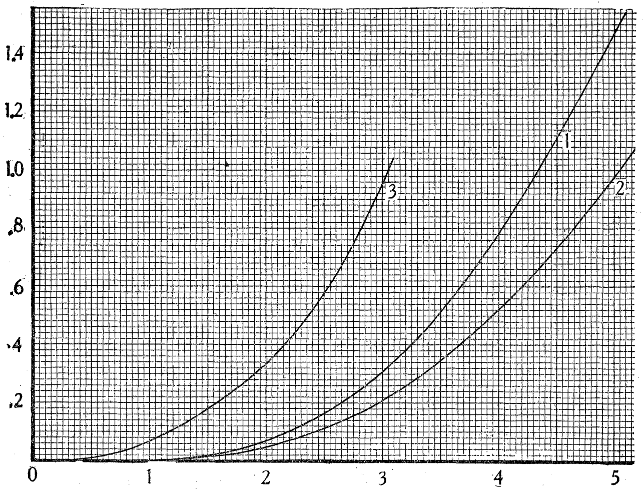

FIGURE 13. Final results, showing the departure from Ohm's law in per cent (ordinates) against current density (abscissae) in 10^6 amp/cm^2. Curve 1 is for silver 2×10^{-5} cm. thick, curve 2 for gold 8×10^{-6} cm. thick, and curve 3 for gold 1.67×10^{-5} cm. thick.

The value of the constant is unity in order to satisfy the condition $\left(\frac{dy}{dx}\right)_{x=0} = 1$. Now the departure from Ohm's law is $\frac{y}{x} - 1$, or

$$\int_0^x \frac{\varphi(x)\,dx}{x}.$$

This may be determined graphically from the experimental curve for φ. The departure from Ohm's law has been computed from the graphs according to this formula for the two thicknesses of gold and for silver, and is reproduced in Figure 13.

THEORETICAL DISCUSSION.

I have already mentioned that J. J. Thomson has suggested that at extreme current densities the current will increase as the square root of the E.M.F., instead of linearly. As far as I know, no theoretical discussion has ever been given of the magnitude of the effect to be expected at lower current densities, where departures are just beginning to be perceptible. Swann [7] has discussed the resistance of thin films from the standpoint of the classical electron theory of conductivity, and has retained the second order terms, subject to the assumption of the classical theory that after every collision of electron with atom all vestige of the effect of the electric force acting on the electron before the collision is wiped out. Under these assumptions he obtains a departure from Ohm's law in the opposite direction from that found experimentally above, or given by Thomson's theory. Subject to the same assumption I have also carried through an examination from the classical point of view, retaining the second and third order terms in the expressions for the velocity imparted to the free electron by the applied force during its free path, and arrive at a result similar to Swann's, that is, a departure from Ohm's law in the drection of a decrease of resistance with high E.M.F.'s. The first term in the departure involves the square of the E.M.F., as considerations of symmetry show that it must. This simple analysis must be incorrect, however. It is not likely that the trend at very high E.M.F.'s found by Thomson is reversed at lower E.M.F.'s, and it is also exceedingly probable that the assumptions at the basis of the classical theory are not exact. One would certainly expect an effect of higher order on the velocity distribution by the applied force, even if there is no effect of zero order.

I have not been able to make the modifications in the classical analysis which would be necessary to take account of terms of higher order in the velocity distribution. I believe that it must involve the details of atomic and crystal structure.

Not being able to give the exact analysis that we could desire, we may fall back on a dimensional analysis. If the mechanism of conduction is a free path mechanism, as is supposed in the classical theory, and as I think most likely, we see that the departure from Ohm's law is going to depend only on the kinematics of the motion of an electron moving with a certain normal velocity in a free path when acceleration is impressed by an additional force from without. This assumes that the number of free electrons is not changed by the external force. The elements that enter our analysis are the acceleration of the electron (α), its mean free path (l), and its normal velocity in the absence of the external impressed force (v). The departure from Ohm's law is to be expressed in terms of these. As we have defined it, the departure from Ohm's law, D, is a ratio of similar quantities, and is hence dimensionless. The only dimensionless combination of α, l, and v is $\frac{\alpha l}{v^2}$. Hence we have

$$D = f\left(\frac{\alpha l}{v^2}\right).$$

Considerations of symmetry show that the unknown function must be an even function of its argument, since reversing the direction of the acceleration leaves the resistance unaltered. Also the acceleration of the electron is determined by its charge, mass, and the applied force by the equation

$$\alpha = \frac{Ee}{m}.$$

Hence finally we have the result

$$D = \text{even function}\left(\frac{Eel}{mv^2}\right).$$

Maxwell assumed, as was perfectly natural, that the function was algebraic, and hence that the first term was proportional to E^2, it being an experimental fact that the constant term vanishes. In the light of the experiments above, however, it is exceedingly questionable whether this assumption is justified. It seems to me that the curve hugs the axis at the origin much more closely than an algebraic curve, and in fact may have contact of an infinitely high order, like an exponential curve. I have not succeeded in satisfactorily reproducing the course of the curve with two or three terms, although this may be done with a fair degree of approximation. I found this significant thing in trying to fit an algebraic curve to the experimental results. If the first term, that is the square term, corresponds to anything real physically, then its magnitude in a three constant formula should not be very different from its value in a two constant formula, the terms of higher order constituting merely correction terms. This is not the case. The square term in a three constant formula fitting the results for 8×10^{-6} gold was only about one fifth as large as in a two constant formula. It seems to be likely, therefore, both from the appearance of the curve and this evidence from computation that the correct form of the curve is not algebraic, or at any rate that the first term is not a square term.

However, merely in order to afford a basis for comparison with previous results, let us assume for the moment that at comparatively small values of the current the most important term is the square term. My results for 8×10^{-6} gold show that at a current density of 10^6 amp/cm^2 the departure from Ohm's law is certainly not any greater than 10^{-4}. Assuming, in lack of anything better, that the coefficient of the square term is unity, and assuming the square law, this would mean that at a current density of 1 amp/cm^2 the departure from Ohm's law is not greater than 10^{-16}. Maxwell stated the limit as one part in 10^{12} for the same current density. The law is probably actually much closer than 10^{-16} at 1 amp/cm^2.

Another argument against the probability of the square law is the very high values that it would mean for the free path. Let us assume again that the coefficient of the square term is unity, and substitute numerical values. We have to put

$$\left(\frac{Eel}{mv^2}\right)^2 = 10^{-4},$$

where E is the electric field at a current of 10^6 amp/cm^2. In accordance with the usual assumption of classical theory, which I have attempted to show elsewhere is very probably true,[8] we put the energy of the electron equal to the energy of a gas molecule at the same temperature (300^0 Abs.). We may simplify by writing $mv^2 = 2k\tau$, where k is the gas constant and τ absolute temperature. We have to solve the above equation for l, the only unknown. The specific resistance of gold is 2.4×10^{-6}. To drive 10^6 amp/cm^2 takes 2.4 volts/cm which is 0.08 Abs. E. S. U. Substituting these values gives

$$l = 4.5 \times 10^{-4} \text{ cm.}$$

It would appear on this basis, therefore, that the free path is of the order of 5×10^{-4} cm. long. This is in the direction that I would like to find the path to differ from the previous results of the classical theory, which gave something of the order of 10^{-8} cm., for my theory demands a long path and few electrons,[8] but I believe that the number above goes rather too far in the desired direction. It is, however, perhaps not impossible that the path should be of the order of 10^{-4} cm. In default of a better theory, my measurements do not afford the basis for a better calculation. It would be necessary to refine on the measurements at current densities less than 10^6 in order to pick out the square term (provided that it exists), or to improve on the theory. I do not believe that the latter will be possible without taking into account the detailed structure of the atom. At the same time, I believe that the existence of the effect at current densities so much lower than would be expected on the basis of a free path of the order of 10^{-8} cm. is at least presumptive evidence that this value of the path is too low.

The appearance of the v^2 term in the formula above suggests an essential observation. The measurements, and the results computed from them, do not refer to metal at the same temperature at the different current densities, but the observations at the higher densities are for the metal at higher temperatures, because of the heating effect. This of course was not eliminated by extrapolating to infinite frequency. The change of temperature at the maximum current density was in the neighborhood of 50° for all specimens. If the form of the function were known, it would have been possible to correct for this temperature effect, but in the absence of the knowledge, I thought it better to give the results as obtained without attempting any correction. If conduction is by a free path mechanism, then at higher velocities the departures from Ohm's law are less than at lower ones, (that is, lower temperatures) so that if the corrections had been applied a deviation from Ohm's law even greater than that shown would have been found at the higher current densities.

We have noticed that the deviations are greater for the thick than for the thin gold. Although the results are not as accurate for the thicker as the thinner leaf, there seems to be no possibility that all the difference can be accounted for by errors of observation. If the free path is long, as I have supposed, an effect in precisely this direction would be expected. For the leaf is considerably less in thickness than the length of the normal path, so that increasing the thickness would have the effect of increasing the average path, and so increasing the departure from Ohm's law.

The departure is considerably less in silver than in gold of the same thickness. This would mean a shorter path in silver than in gold. In view of the greater conductivity of silver this may mean that the number of free electrons is greater in silver than in gold. It is, however, perhaps dangerous to drive the comparison too far between the different metals, because the much closer approach to normal of both the specific resistance and the temperature coefficient of resistance of silver suggests that the internal conditions may not be comparable.

SUMMARY.

By the employment of a new method, by which the resistance is measured simultaneously for a heavy direct current and a small superposed alternating current of acoustical frequency, deviations from Ohm's law at high current densities have been detected and measured in leaf gold and silver. The maximum current densities were about 5×10^6 amp/cm^2, and the deviations from Ohm's law were of the order of one per cent. If the mechanism of conduction is a free path mechanism, these results probably mean that the free path is much longer than supposed on the classical electron theory.

It is a pleasure to acknowledge my indebtedness to my assistant Mr. J. C. Slater for making nearly all the readings.

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